CE-089 Manning Equation for Open Channels

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Manning Equation - Open Channel Flow Calculations

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COURSE CONTENT

1. Introduction

The Manning equation is a widely used empirical equation for uniform open channel flow of water. It provides a relationship among several open channel flow parameters of interest: flow rate or average velocity, bottom slope of the channel, cross-sectional area of flow, wetted perimeter, and Manning roughness coefficient for the channel. Open channel flow takes place in natural channels like rivers and streams, as well as in manmade channels like those used to transport wastewater and in circular sewers flowing partially full.

The main topic of this course is uniform open channel flow, in which the channel slope, water velocity and water depth remain constant. This includes a variety of example calculations with the Manning equation and the use of Excel spreadsheets for those calculations.

Figure 1. Bighorn River in Montana – a Natural Open Channel

Image Source: National Park Service, Bighorn Canyon National Recreational Area website at: https://www.nps.gov/bica/planyourvisit/bighorn-river-in-montana.htm

Figure 2. Irrigation Canal Branch in Sinai – A man-made open channel

Image Source: Egypt-Finland Agric. Res Proj

2. Learning Objectives

At the conclusion of this course, the student will

- Know the difference between laminar & turbulent open channel flow.
- Know the difference between steady state & unsteady state open channel flow.
- Know the difference between uniform & non-uniform open channel flow.
- Be able to calculate the hydraulic radius for flow of a specified depth in an open channel with specified cross-sectional shape and size.

- Be able to calculate the Reynolds Number for a specified open channel flow and determine whether the flow will be laminar or turbulent flow.

- Be able to use tables such as the examples given in this course to determine a value for Manning roughness coefficient for flow in a manmade channel.

- Be able to use the Manning Equation to calculate volumetric flow rate, average velocity, Manning roughness coefficient, or channel bottom slope, if given adequate information about a reach of open channel flow

- Be able to use the Manning Equation, with an iterative procedure, to calculate normal depth for specified volumetric flow rate, channel bottom slope, channel shape & size, and Manning roughness coefficient for a reach of open channel flow

- Be able to make Manning Equation calculations in either U.S. units or S.I. units

- Be able to calculate the Manning roughness coefficient for a natural channel based on descriptive information about the channel.

- Be able to use the Manning Equation to make calculations for the flow of water in a circular pipe either flowing full or flowing half full.

- Be able to carry out a variety of calculations for full or partially full flow of water under gravity in a circular pipe site.

3. Topics Covered in this Course

I. Open Channel Flow vs Pipe Flow

II. Classifications of Open Channel Flow

A. Laminar or Turbulent Flow

4. Open Channel Flow vs Pipe Flow

The term "open channel flow" is used to refer to flow with a free surface at atmospheric pressure, in which the driving force for flow is gravity. Pipe flow, on the other hand is used to refer to flow in a closed conduit under pressure, in which the primary driving force is typically pressure. Open channel flow occurs in natural channels, such as rivers and streams and in manmade channels, as for storm water, waste water and irrigation water. This course is about open channel flow, and in particular, about uniform open channel flow. The next section covers several different classifications of types of open channel flow, including clarification of the difference between uniform and nonuniform open channel flow.

5. Classifications of Open Channel Flow

A. Turbulent and Laminar Flow: Description of a given flow as being either laminar or turbulent is used for several fluid flow applications (like pipe flow and flow past a flat plate) as well as for open channel flow. In each of these fluid flow applications a Reynolds number is used for the criterion to determine whether a given flow will be laminar or turbulent. For open channel flow a Reynolds number below 500 is typically used as the criterion for laminar flow, while the flow will typically be turbulent for a Reynolds number greater than 12,500. For a flow with Reynolds number between 500 and 12,500, other conditions, like the upstream channel conditions and the roughness of the channel walls will determine whether the flow is laminar or turbulent.

Background on Laminar and Turbulent Flow: Osborne Reynolds reported in the late 1800s on experiments that he performed observing the difference between laminar and turbulent flow in pipes and quantifying the conditions for which each would occur. In his classic experiments, he injected dye into a transparent pipe containing a flowing fluid. He observed that the dye flowed in a streamline and didn't mix with the rest of the fluid under some conditions, which he called laminar flow. Under other conditions, however, he observed that the net velocity of the fluid was in the direction of flow, but there were eddy currents in all directions that caused mixing of the fluid. Under these turbulent flow conditions, the entire fluid became colored with the dye. The figure below illustrates laminar and turbulent open channel flow.

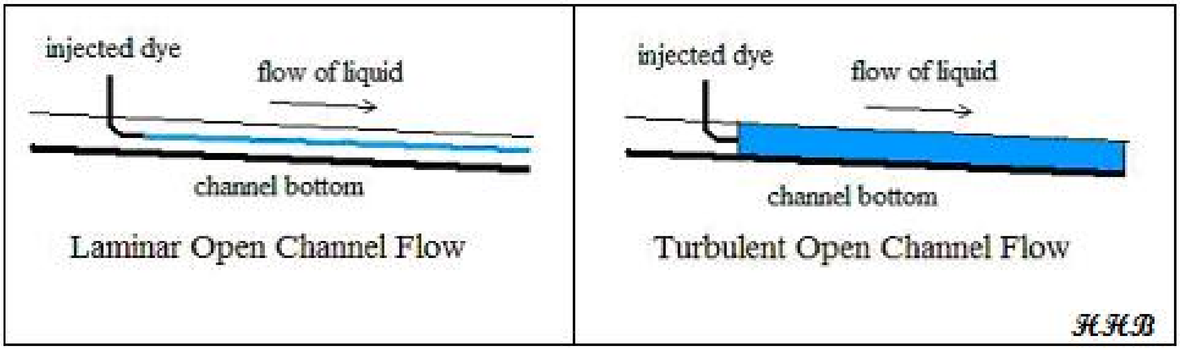


Figure 3. Dye injection into laminar & turbulent open channel flow

Laminar flow, sometimes also called streamline flow, occurs for flows with high fluid viscosity and/or low velocity. Turbulent flow takes place for flows with low fluid viscosity and/or high velocity.

More discussion of the Reynolds number and its calculation for open channel flow are given in Section 6 of this course. For most practical cases of water transport in either manmade or natural open channels, the Reynolds number is greater than 12,500, and thus the flow is turbulent. One notable exception is flow of a thin liquid layer on a large flat surface, such as rainfall runoff from a parking lot, highway, or airport runway. This type of flow, often called sheet flow, is typically laminar.

B. <u>Unsteady State and Steady State Flow:</u> The concepts of steady state and unsteady state flow are used for a variety of fluid flow applications, including open channel flow. Steady state flow is taking place whenever there are no changes in velocity pattern or magnitude with time at a given channel cross section. When unsteady state flow is present, however, there are changes of velocity with time at any given cross section in the flow. Steady state open channel flow takes place when a constant flow rate of liquid is passing through the channel. Unsteady state open channel flow takes place when there is a changing flow rate, as for example in a river after a rainstorm. Steady state or nearly steady state conditions are present for many practical open channel flow situations. The equations and calculations presented in this course are all for steady state flow.

C. <u>Critical, Subcritical, and Supercritical Flow:</u> Any open channel flow must be one of these three classifications: supercritical, subcritical or critical flow. The interpretation of these three classifications of open channel flow, and the differences among them, aren't as obvious or intuitive as the interpretation and the differences for the other classifications (steady and unsteady state, laminar and turbulent, and uniform and non-uniform flow). Some of the behaviors for subcritical and supercritical flow and the transitions between them may not be what you would intuitively expect. Supercritical flow takes place when there is a relatively high liquid velocity and relatively shallow depth of flow. Subcritical flow, as one might expect, takes place when there is a relatively low liquid velocity and relatively deep flow. The Froude number ($Fr = V/(gl)^{1/2}$) provides information about whether a given flow is supercritical, subcritical or critical. For subcritical flow, Fr is less than one; for supercritical flow, it is greater than one; and for critical flow it is equal to one. Further details about subcritical, supercritical and critical flow are beyond the scope of this course.

D. <u>Non-Uniform and Uniform Flow:</u> Uniform flow will occur in a reach of open channel whenever there is a constant flow rate of liquid passing through the channel, the bottom slope is constant, the channel surface roughness is constant, and the cross-section shape & size are constant. Under these conditions, the depth of flow and the average velocity of the flowing liquid will remain constant in that reach of channel. Non-uniform flow will be present for reaches of channel where there are changes in the bottom slope, channel surface roughness, cross-section shape, and/or cross-section size. Whenever the bottom slope, surface roughness, and channel cross-section shape and size become constant in a downstream reach of channel, another set of uniform flow conditions will occur there. This is illustrated in Figure 4.

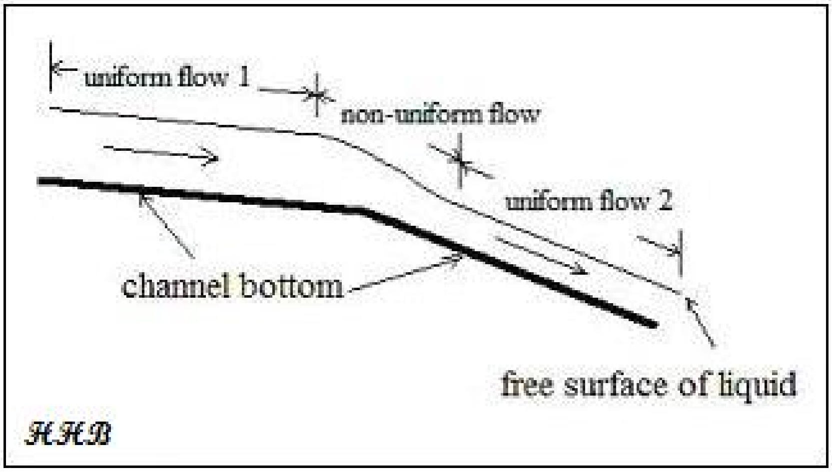


Figure 4. Non-uniform and Uniform Open Channel Flow

6. Manning Equation/Uniform Open Channel Flow Basics

As just described above, uniform open channel flow takes place in a channel reach that has constant channel cross-section size and shape, constant surface roughness, and constant bottom slope, with a constant volumetric flow rate of liquid passing through the channel. These conditions lead to flow at a constant depth of flow and constant liquid velocity, as illustrated in Figure 2.

A. The Manning Equation is an empirical equation that was developed by the French engineer, Philippe Gauckler in 1867. It was redeveloped by the Irish engineer, Robert Manning, in 1890. Although this equation is also known as the Gauckler-Manning equation, it is much more commonly known simply as the Manning equation or Manning formula in the United States. This formula gives the relationship among several parameters of interest for uniform flow of water in an open channel. Not only is the Manning equation empirical, it is also a dimensional equation. This means that the units to be used for each of the parameters must be specified for a given constant in the equation. For commonly used U.S. units the Manning Equation and the units for its parameters are as follows:

$$Q = (1.49/n)A(R_h^{2/3})S^{1/2} \qquad (1)$$

Where:

- Q = the volumetric flow rate of water passing through the channel reach in ft^3/sec.

- n = the Manning roughness coefficient for the channel surface (a dimensionless, empirical constant).

- A = the cross-sectional area of water normal to the flow direction in ft^2.

- R_h = the hydraulic radius (R_h = A/P). (A is the cross-sectional area as defined just above in ft^2 and P is the wetted perimeter of the cross-sectional area of flowing water in ft.

- S = the bottom slope of the channel* in ft/ft (dimensionless).

*S is actually the slope of the energy grade line. For uniform flow, however, the depth of flow is constant and the velocity head is constant, so the slope of the energy grade line is the same as that of the hydraulic grade line and is the same as the slope of the water surface, which is the same as the channel bottom slope. For convenience, the channel bottom slope is typically used for S in the Manning Equation.

B. <u>Manning Roughness Coefficient, n,</u> is a dimensionless, empirical constant, as just described above. Its value depends on the nature of the channel and its surfaces. There are tables with values of n for various channel types and surfaces in many handbooks and textbooks, as well as at several online sources. Table 1 below is a typical table of this type. This table gives n values for several manmade open channel surfaces. Values of n for natural channels will be addressed in Section 8.

Table 1. Manning Roughness Coefficient, n, for Selected Surfaces

Channel Surface	Manning Roughness Coefficient, n
Asbestos cement	0.011
Brass	0.011
Brick	0.015
Cast-iron, new	0.012
Concrete, steel forms	0.011
Concrete, wooden forms	0.015
Concrete, centrifugally spun	0.013
Copper	0.011
Corrugated metal	0.022
Galvanized Iron	0.016
Lead	0.011
Plastic	0.009
Steel - Coal-tar enamel	0.01
Steel - New unlined	0.011
Steel - Riveted	0.019
Wood stave	0.012

Source for n values in Table 1: http://www.engineeringtoolbox.com

C. <u>The Reynolds number</u> for open channel flow is defined as $\mathbf{Re = \rho V R_h/\mu}$**,** where $\mathbf{R_h}$ is the hydraulic radius, as defined above, **V** is the liquid velocity (= Q/A), and ρ and μ are the density and viscosity respectively of the flowing fluid. Since the Reynolds number is dimensionless, any consistent set of units can be used for $\mathbf{R_h}$**, V,** $\boldsymbol{\rho}$**, and** $\boldsymbol{\mu}$**.** If done properly, all of the units will cancel out, leaving Re dimensionless.

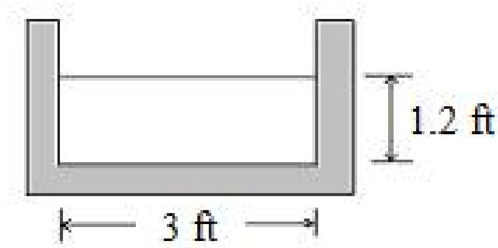
The flow must be in the turbulent regime in order to use the Manning equation for uniform open channel flow. It is fortunate that nearly all practical instances of water transport through an open channel have Re greater than 12,500, in which case the flow is turbulent,and the Manning equation can be used.

The Manning equation is specifically for the flow of water, and no water properties are required in the equation. In order to calculate the Reynolds number to check on whether the flow is turbulent, however, values of density and viscosity for the water in question are needed. Tables of density and viscosity values for water as a function of temperature are available in many textbooks, handbooks, and websites. Table 2 below gives values for density and viscosity of water from 32°F to 70°F.

Table 2. Density and Viscosity of Water

Temperature, °F	Density, slugs/ft^3	Dynamic Viscosity, lb-s/ft^2
32	1.940	3.732×10^{-5}
40	1.940	3.228×10^{-5}
50	1.940	2.730×10^{-5}
60	1.938	2.334×10^{-5}
70	1.936	2.037×10^{-5}

Example #1: Water at 60ºF is flowing 1.2 feet deep in a 3 foot wide rectangular open channel, as shown in the diagram below. The channel is made of concrete (made with wooden forms) and has a bottom slope of 0.0008. Determine whether the flow is laminar or turbulent.



Solution: Based on the problem statement, this will be uniform flow. The flow is probably turbulent, however the velocity is needed in order to calculate the Reynolds number. Hence we will assume that the flow is turbulent, use the Manning equation to calculate Q and V. Then Re can be calculated to check on whether the flow is indeed turbulent.

The parameters needed for the right side of the Manning equation are as follows:

From Table 1, for concrete made with wooden forms: $n = 0.015$

$$A = (1.2)(3) = 3.6 \text{ ft}^2$$

$$P = 3 + (2)(1.2) = 5.4 \text{ ft}$$

$$R_h = A/P = 3.6/5.4 = 0.6667 \text{ ft}$$

$S = 0.0008$ (given in problem statement)

Substituting into the Manning equation ($\mathbf{Q = (1.49/n)A(R_h^{2/3})S^{1/2}}$) :

$Q = 1.49/0.015)(3.6)(0.6667^{2/3})(0.0008^{1/2}) = 7.72$ cfs

Now the average velocity, V, can be calculated:

$V = Q/A = 7.72/3.6 = 2.14$ ft/sec

From Table 2, for 60°F: $\rho = 1.938$ slugs/ft^3 and $\mu = 2.334 \times 10^{-5}$ lb-sec/ft^2

Substituting into **$Re = \rho V R_h/\mu$:**

$Re = (1.938)(2.14)(0.6667)/2.334 \times 10^{-5} = 118{,}470$

Since Re > 12,500, this open channel flow is turbulent

D. Hydraulic Radius is a parameter that must be calculated for various channel shapes in order to use the Manning Equation. Some common cross-sectional shapes used for open channel flow are **rectangular, trapezoidal triangular, circular, and semicircular**. Formulas for the hydraulic radius for each of these channel shapes will now be presented.

A **rectangular** channel allows easy calculation of the hydraulic radius. The bottom width will be represented by b and the depth of flow will be represented by y, as shown in the Figure 5 below.

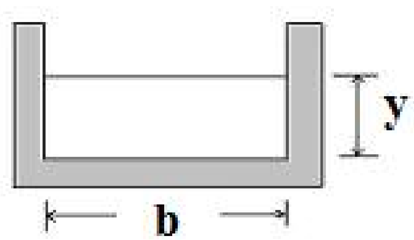


Figure 5. Rectangular Open Channel Cross-Section

The area and wetted perimeter will be as follows:

$A = by$ $\qquad\qquad$ $P = 2y + b$

Then $R_h = A/P$, or:

For a rectangular channel: $\mathbf{R_h = by/(2y + b)}$ (2)

A **trapezoidal** cross-section is used for some manmade open channels and can be used as an approximation of the cross-sectional shape for some natural channels. Figure 6 shows the parameters typically used to describe the size and shape of a trapezoidal channel.

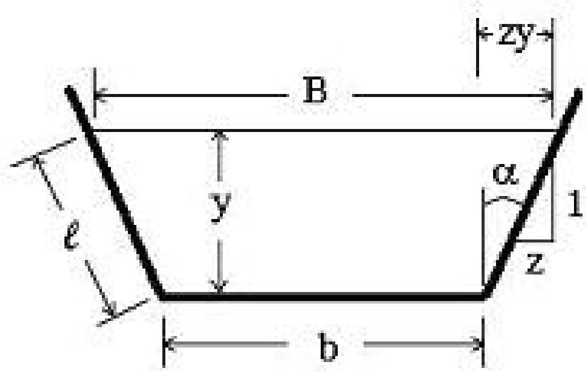


Figure 6. Trapezoidal Open Channel Cross-section

The channel bottom width and the water depth are represented by **b** and **y**, the same as with the rectangular channel. Additional parameters for the trapezoidal channel shape are:

- **B**, the water surface width;
- ***ℓ*,** the wetted length of the sloped side;
- α, the angle of the sloped side of the channel from the vertical; and

- **z**, the channel side slope expressed as horiz**:**vert = z**:**1.

The size and shape of a trapezoidal channel are often specified with the bottom width, **b**, and the side slope, **z**. The hydraulic radius for flow in a trapezoidal open channel can be expressed in terms of **y**, **b**, and **z**, as follows:

i) The cross-sectional area of flow, **A**, is the area of the trapezoid in Figure 4:

$$A = y(b + B)/2 \;=\; (y/2)(b + B)$$

From Figure 6, it can be seen that the surface width, B, is greater than the bottom width, b, by zy at each end, or:

$$B = b + 2zy$$

Substituting for B into the equation for A gives:

$$A = (y/2)(b + b + 2zy) \;=\; (y/2)(2b + 2zy)$$

Simplifying gives: $\mathbf{A = by + zy^2}$

As shown in Figure 6, the wetted perimeter of the cross-sectional area of flow is

$$P = b + 2\ell$$

By Pythagoras' Theorem for the triangle at each end of the trapezoid:

$$\ell^2 = y^2 + (yz)^2 \quad \text{or} \quad \ell = [y^2 + (yz)^2]^{1/2}$$

Substituting for ℓ into the equation for P and simplifying gives:

$$\mathbf{P = b + 2y(1 + z^2)^{1/2}}$$

Substituting for A and P in $R_h = A/P$ gives:

For a **trapezoidal** open channel: $\mathbf{R_h = (by + zy^2)/[b + 2y(1 + z^2)^{1/2}]}$ (3)

Example #2: Determine the hydraulic radius of water flowing 1.5 ft deep in a trapezoidal open channel with a bottom width of 2 ft and side slope of horiz**:**vert = 3**:**1.

Solution: From the problem statement, y = 1.5 ft, b = 2 ft, and z = 3. Substituting these values into the expression for hydraulic radius gives:

$\mathbf{R_h} = (2*1.5 + 3*1.5^2)/[2 + 2*1.5(1 + 3^2)^{1/2}] = \underline{\mathbf{0.849\ ft}}$

This type of calculation can conveniently be done using an Excel spreadsheet like the simple one shown in the screenshot in Figure 7 below. This particular spreadsheet is set up to allow user entry of the channel bottom width, the depth of flow, and the side slope expressed as z. The spreadsheet then calculates the cross-sectional area of flow, **A**, the wetted perimeter, **P**, and the hydraulic radius, $\mathbf{R_h}$, for the trapezoidal channel. The equations shown at the bottom of the worksheet are the same as those presented and discussed in this course.

Hydraulic Radius Calculator (U.S. units)

II. Trapezoidal Channel:

Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*

Inputs			**Calculations**		
Bottom width, **b** =	2	ft	Cross-Sect. Area, **A** =	9.8	ft^2
Depth of flow, **y** =	1.5	ft	Wetted Perimeter, **P** =	11.5	ft
Side Slope, **z** = (H:V = z:1)	3		Hydraulic Radius, **R** =	0.849	ft

Equations used for calculations:

$A = by + zy^2$ (cross-sectional area - trapezoidal channel)

$P = b + 2y(1 + z^2)^{1/2}$ (wetted perimeter - trapezoidal)

$R = A/P$ (hydraulic radius)

Figure 7. Hydraulic Radius Calculator Spreadsheet

The **triangular** open channel cross-sectional shape is the third one that we'll be considering. Figure 8 below shows the parameters typically used to specify the size and shape of a triangular channel. They are:

- **B**, the surface width of the water in the channel
- **y**, the water depth in the channel, measured from the triangle vertex

- ℓ, the wetted length of the sloped side; and
- **z**, the channel side slope expressed as horiz**:**vert = z**:**1.

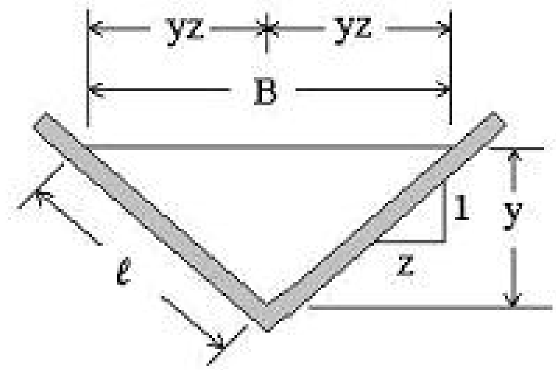


Figure 8. Triangular Open Channel Cross-Section

For a triangular open channel, it's convenient to have the hydraulic radius expressed in terms of y and z, which can be done as follows:

The area of the triangle in Figure 8, which represents the area of flow is:
A = (1/2)By, but as shown in the figure, B = 2zy. Substituting for B in the equation for A and simplifying gives:

$$\mathbf{A = y^2z}$$

Also from Figure 8, it can be seen that the wetted perimeter is: P = 2ℓ.

Substituting $\ell = [y^2 + (yz)^2]^{1/2}$ (as shown above for the trapezoid), and simplifying gives:

$$\mathbf{P = 2[y^2(1 + z^2)]^{1/2}}$$

Substituting for A and P in $R_h = A/P$ gives:

For a **trianglular** open channel: $\mathbf{R_h = y^2z/\{2[y^2(1 + z^2)]^{1/2}\}}$ (4)

Circular pipes are used for open channel (gravity) flow for applications like storm sewers, sanitary sewers, and circular culverts. These pipe typically flow only partially full most of the time, but the full flow scenario is often used for hydraulic design. Hydraulic radius expressions for the full flow and half full cross-sections will be developed here. There will be additional discussion of partially full pipe flow in Section 7.

Figure 9 shows a diagram for a pipe flowing full and a pipe flowing half full. The only parameters needed for either of these cases are the diameter and the radius of the pipe.

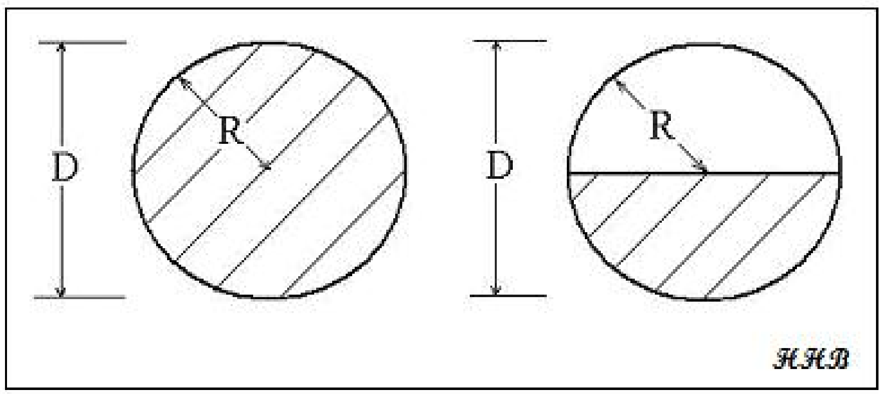


Figure 9. Circular and semicircular Open Channel Cross-Sections

The hydraulic radius for a circular pipe of diameter D, flowing full, can be calculated as follows:

The cross-sectional area of flow is: $A = \pi D^2/4$

The wetted perimeter is: $P = \pi D$

The hydraulic radius is: $R_h = A/P = (\pi D^2/4)/(\pi D)$

Or simply (for a pipe flowing full): $\mathbf{R_h = D/4}$ (5)

For the semicircular shape of a pipe flowing exactly half full, the area, A, and the wetted perimeter, P, will each be half of the values for the full pipe flow. Thus the hydraulic radius will remain the same, so

(for a pipe flowing half full): $$\mathbf{R_h = D/4} \quad (6)$$

E. The Manning Equation in SI Units is the same as that for U.S. units except that the constant is 1.00 instead of 1.49:

$$\mathbf{Q = (1.00/n)A(R_h^{2/3})S^{1/2}} \quad (7)$$

Where:

- Q = the volumetric flow rate of water passing through the channel reach in m^3/s.

- n = the Manning roughness coefficient for the channel surface (a dimensionless, empirical constant).

- A = the cross-sectional area of water normal to the flow direction in m^2.

- R_h = the hydraulic radius in m (R_h = A/P). (A is the cross-sectional area as defined just above in m^2 and P is the wetted perimeter of the cross-sectional area of flowing water in m.

- S = the bottom slope of the channel* in m/m (dimensionless).

F. The Manning Equation in terms of Average Velocity: For some calculations, it is better to have the Manning Equation expressed in terms of average velocity, V, instead of in terms of volumetric flow rate. The definition of average velocity is $\mathbf{V = Q/A}$, where Q and A are as previously defined. Substituting Q = VA into the Manning equation as given in Equation (1), and solving for V gives the following form of the Manning equation.

For U.S. units: $$\mathbf{V = (1.49/n)(R_h^{2/3})S^{1/2}} \quad (8)$$

For S.I. units, the constant is 1.00 instead of 1.49, giving:

For S.I. units: $$\mathbf{V = (1.00/n)(R_h^{2/3})S^{1/2}} \qquad (9)$$

7. Manning Equation Calculations for Manmade Channels

A. <u>The Easy Parameters to Calculate with the Manning Equation:</u> Several different parameters can be the "unknown" to be calculated with the Manning equation, based on known values for enough other parameters. If Q and V, S, or n is the unknown parameter to be calculated, and enough information is known to calculate the hydraulic radius, then the solution involves simply substituting values into the Manning equation and solving for the desired unknown parameter. These four parameters are thus the "easy parameters to calculate." This type of Manning equation calculation is illustrated with several examples here. Then in the next section, we'll take a look at the hard parameter to calculate, normal depth.

Example #3: Use the Manning equation to determine the volumetric flow rate and average velocity of water flowing 0.9 m deep in a trapezoidal open channel with bottom width equal to 1.2 m and side slope of horiz**:**vert = 2**:**1. The channel is concrete poured with steel forms and its bottom slope is 0.0003.

Solution: The hydraulic radius can be calculated from the specified information (y = 0.9 m, b = 1.2 m, & z = 2) using the formula for a trapezoidal channel as follows:

$$\mathbf{R_h = (by + zy^2)/[b + 2y(1 + z^2)^{1/2}]}$$

$$= (1.2*0.9 + 2*0.9^2)/[1.2 + 2*0.9(1 + 2^2)^{1/2}]$$

$R_h = 0.517$ m   (also: $A = by + zy^2 = 1.2*0.9 + 2*0.9^2 = 2.70$ m^2

Substituting R_h and A into Equation (1) along with S = 0.0003 (given) and n = 0.011 (from Table 1) gives:

$\mathbf{Q = (1.00/n)A(R_h^{2/3})S^{1/2}}$ = $(1.00/0.011)(2.70)(0.517^{2/3})(0.0003^{1/2})$

$\underline{Q = 2.74 \text{ m}^3/\text{s}}$

Now the average velocity, V, can be calculated from V = Q/A = 2.74/2.70 m/s

$\underline{V = 1.01 \text{ m/s}}$

This type of calculation is also easy to make with an Excel spreadsheet, like the one shown in the Figure 8 screenshot on the next page.

Example #4: What would be the required slope for a 15 inch diameter circular storm sewer made of centrifugally spun concrete, if it needs to have an average velocity of at least 3.0 ft/sec when it's flowing full?

Solution: For the 15" diameter sewer, $R_h = D/4 = (15/12)/4 = 0.3125$ ft. From Table 1, for centrifugally spun concrete, n = 0.013. Substituting these values for R_h and n, along with the given value of V = 3.0 ft/sec, into Equation (8) and solving for S gives:

$S = \{(0.013)(3.0)/[1.49(0.3125)^{2/3}]\}^2$ = **$\underline{0.003231 = S}$**

Calculation of discharge, Q, and average velocity, V (S.I. Units)					
Using the Manning Equation for Uniform Open Channel Flow					
II. Trapezoidal Channel Calculations:					
Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*					
Inputs			**Calculations**		
Bottom width, **b** =	1.2	m	Cross-Sect. Area, **A** =	2.70	m^2
Depth of flow, **y** =	0.9	m	Wetted Perimeter, **P** =	5.2	m
Side Slope, **z** = (H:V = z:1)	2		Hydraulic Radius, **R** =	0.517	m
Manning roughness, **n** =	0.011		Discharge, **Q** =	2.74	m^3/s
Channel bottom slope, **S** =	0.0003	m/m	Ave. Velocity, **V** =	1.01	m/s

Figure 10. A Spreadsheet for Q & V in a Trapezoidal Channel

Determination of the required Manning roughness coefficient, **n**, for a specified flow rate or velocity, bottom slope, and adequate information to calculate the hydraulic radius, would be a less common calculation, but would proceed in a manner very similar to Example #3 and Example #4.

B. <u>The Hard Parameter to Calculate - Determination of Normal Depth:</u>
When the depth of flow, y, is the unknown parameter to be determined using the Manning equation, an iterative calculation procedure is often required. This is because an equation with y as the only unknown can typically be obtained, but

the equation usually can't be solved explicitly for y, making this "the hard parameter to calculate." The depth of flow for a given flow rate through a channel reach of known shape size & material and known bottom slope is called the **normal depth,** and is sometimes represented by the symbol, y_o.

The typical situation requiring determination of the normal depth, y_o, will have specified values for the flow rate, Q, the Manning roughness coefficient, n, and channel bottom slope, S, along with adequate channel size and shape information to allow A and R_h to be expressed as functions of y_o.

The approach for calculating the normal depth, y_o, for a situation as described above, is to rearrange the Manning equation to:

$$\mathbf{AR_h^{2/3} = Qn/(1.49S^{1/2})} \qquad (10)$$

The right side of this equation will be a constant and the left side will be an expression with y_o as the only unkown. The next couple of examples illustrate calculation of y_o using an iterative calculation with Equation (10).

Example #5: Determine the normal depth for a water flow rate of 20 ft^3/sec, through a rectangular channel with a bottom slope of 0.00025, bottom width of 4 ft, and Manning roughness coefficient of 0.012.

Solution: Substituting the expressions for A and R_h for a rectangular channel into the left hand of Equation (10) and substituting the given values for Q, n, and S into the right side, gives:

$$4y_o(4y_o/(4 + 2y_o))^{2/3} = (20)(0.012)/[1.49\ (0.00025^{1/2})] = 10.187$$

This equation has y_o as the only unknown. The equation can't be solved explicitly for y_o, but it can be solved by an iterative (trial and error) process as illustrated in the Excel spreadsheet screenshot in Figure 11 on the next page. The spreadsheet screenshot shows the solution to be: $\mathbf{y_o = 2.40}$ **ft**, accurate to 3 significant figures. Note that this type of iterative calculation can also be accomplished with Excel's Goal Seek or Solver tool.

Calculation of Normal Depth (y_o) for Uniform Open Channel Flow

Using the Manning Equation (U.S. units)

I. Rectangular Channel:

Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*

Inputs

Bottom width, **b** =	4	ft
Manning roughness, **n** =	0.012	
Channel bottom slope, **S** =	0.00025	ft/ft
Volumetric Flow Rate, **Q** =	20	cfs

The Manning equation can be rearranged to: $Q^*n/(1.49^*S^{1/2}) = A^*R^{2/3}$

Calculations

$Q^*n/(1.49^*S^{1/2})$ = 10.187 ft^2

(= $A^*R^{2/3}$ = target value for iterative calculation)

For a rectangular channel: $A^*R^{2/3}$ = $(by_o)^*[by_o/(b + 2y_o)]^{2/3}$

Iterative (trial & error) Solution:

y_o ,ft	$A^*R^{2/3}$
1	3.053
2	8.000
3	13.551
2.5	10.728
2.4	10.173
2.41	10.229
	0.000
	0.000

NOTE: In this example the value of y_o, correct to 3 significant figures is 2.40 ft, because 10.173 is as close as we can get to the target value of 10.187 with 3 significant figures for y_o.

Figure 11. A Spreadsheet for Normal Depth in a Rectangular Channel

Example #6: Determine the normal depth for a water flow rate of 20 ft^3/sec, through a trapezoidal channel with a bottom slope of 0.00025, bottom width of 4 ft, side slope of horiz**:**vert = 2**:**1, and Manning roughness coefficient of 0.012.

Solution: The values of Q, n, & S are the same as for Example #5, so the right hand side of Equation (10) will remain the same at 10.187. The left hand side will be somewhat more complicated with the expression for R_h as a function of y_o for a trapezoid. Equation (10) for this calculation is:

$$(4y_o + 2y_o^2)\{(4y_o + 2y_o^2)/[4 + 2y_o(1 + 2^2)^{1/2}]\}^{2/3} = 10.187$$

The iterative calculations leading to **y_o = 1.49 ft** are shown below. The solution is y_o = 1.49 ft, because 10.228 is closer to the target value of 10.187, than the value of 10.094 for y_o = 1.48 or 10.363 for y_o = 1.50.

y_o ,ft	1	2	1.5	1.4	1.49	1.48
$A*R^{2/3}$	4.767	18.428	10.363	9.056	10.228	10.094

C. Circular Pipes Flowing Full : Because of the simple form of the equations for hydraulic radius and cross-sectional area as functions of the diameter for a circular pipe flowing full ($R_h = D/4$ and $A = \pi D^2/4$), the Manning equation can be conveniently used to calculate Q and V, D, S, or n if the other parameters are known. Several useful forms of the Manning equation for a circular pipe flowing full under gravity are:

$$\mathbf{Q = (1.49/n)(\pi D^2/4)((D/4)^{2/3})S^{1/2}} \qquad (11)$$

$$\mathbf{V = (1.49/n)((D/4)^{2/3})S^{1/2}} \qquad (12)$$

$$\mathbf{D = 4[Vn/(1.49S^{1/2})]^{3/2}} \qquad (13)$$

$$\mathbf{D = 1.33Qn/S^{1/2}} \qquad (14)$$

Note that these four equations are for the U.S. units previously specified. For S.I. units, the 1.49 should be replaced with 1.00 in the first three equations. In Equation (14), 1.33 should be replaced with 0.893.

Hydraulic design of storm sewers is typically based on full pipe flow using equations (11) through (14).

Example #7: What would be the flow rate and velocity in a 30 inch diameter storm sewer that has n = 0.011 and slope = 0.00095, when it is flowing full under gravity?

Solution: Substituting the given values of n, D, and S into Equation (12) gives:

$$V = (1.49/0.011)[((30/12)/4)^{2/3}](0.00095^{1/2}) = \underline{\mathbf{3.052\ ft/sec = V}}$$

Then Q can be calculated from $Q = VA = V(\pi D^2/4)$

$$Q = (3.05\ \text{ft/sec})[\pi(2.5^2)/4\ \text{ft}^2] = \underline{\mathbf{15.0\ cfs = Q}}$$

D. <u>Circular Pipes Flowing Partially Full :</u> Although hydraulic design of storm sewers is typically done on the basis of the circular pipe flowing full, a storm sewer will often flow partially full due to a storm of intensity less than the design storm. Thus, there is sometimes interest in calculations for partially full pipe flow, such as the flow rate or velocity at a given depth of flow or the depth of flow for a given velocity or flow rate.

Figure 12 shows the depth of flow, y, and the diameter, D, as used for partially full pipe flow calculations.

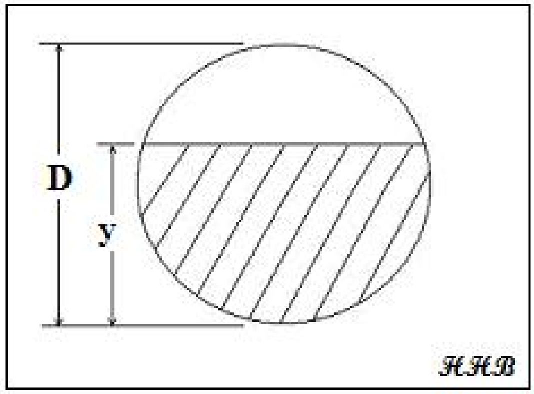


Figure 12. Depth of flow, **y**, and Diameter, **D**, for Partially Full Pipe Flow

Graphical Solution: One common way of handling partially full pipe flow calculations is through the use of a graph that correlates V/V_{full} and Q/Q_{full} to y/D, as shown in Figure 13.

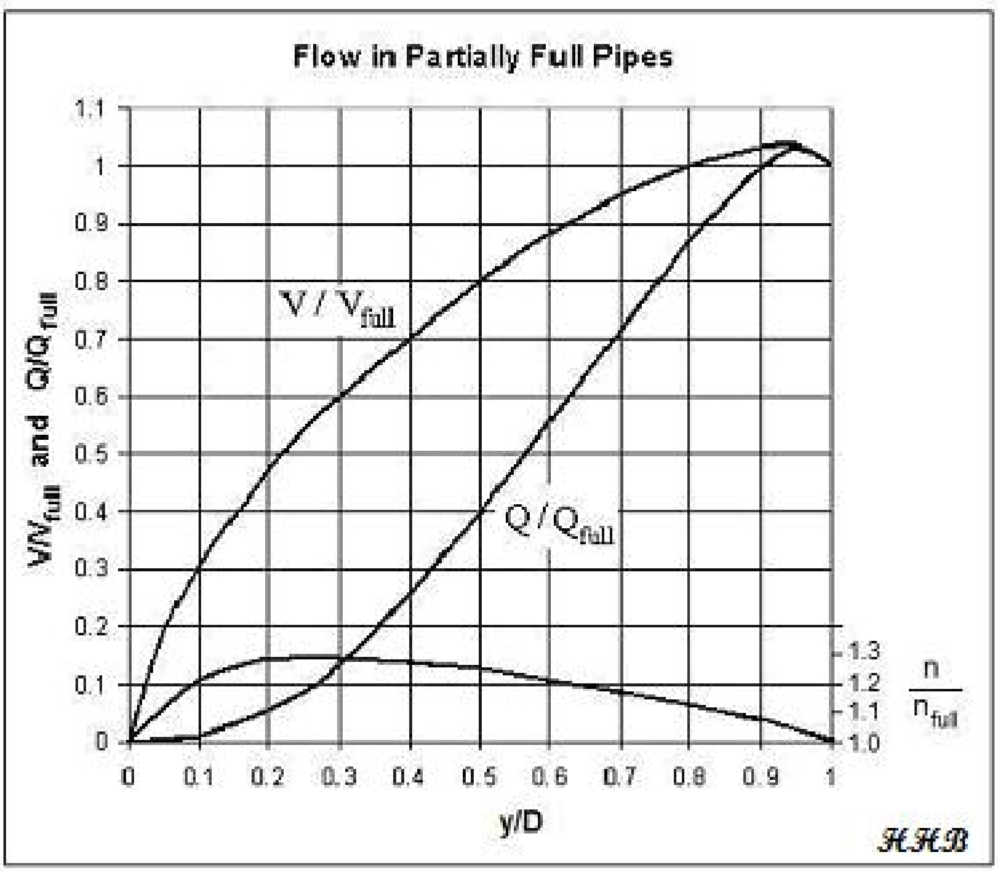


Figure 13. Flow Rate and Velocity Ratios in Pipes Flowing Partially Full

If values of D, V_{full} and Q_{full} are known or can be calculated, then the velocity, V, and flow rate, Q, can be calculated for any depth of flow, y, in that pipe through the use of figure 13.

Example #8: What would be the velocity and flow rate in the storm sewer of Example #7 (D = 30", n = 0.011, S = 0.00095) when it is flowing at a depth of 12 inches?

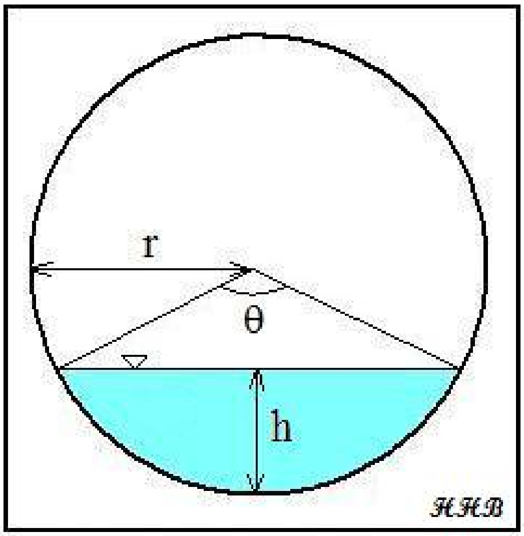
Solution: From the solution to Example #7: V_{full} = 3.052 ft/sec and Q_{full} = 15.0 cfs. From the given y and D values: y/D = 12/30 = 0.40. From Figure 13, for y/D = 0.40, V/V_{full} = 0.70 and Q/Q_{full} = 0.25.

$$V = (V/V_{full})V_{full} = (0.70)(3.052)\text{ ft/sec} = \underline{\mathbf{2.14\text{ ft/sec} = V}}$$

$$Q = (Q/Q_{full})Q_{full} = \underline{\mathbf{(0.25)(15.0)\text{ cfs} = Q}}$$

Background on Equations for Partially Full Pipe Flow: There are equations available to calculate the A and P for any depth of flow in a circular pipe (as presented below). These equations allow calculation of the hydraulic radius for partially full pipe flow. If the hydraulic radius calculated by this method is used with the Manning equation, using the full pipe value for n, the calculated flow rate and velocity don't agree well with experimental measurements. This was observed by T.R Camp in 1946 (reference #3). Camp developed a method that uses Manning roughness, n, to be variable as a function of y/D, which makes calculated results agree with experimental measurements. Camp is the original source for a diagram like Figure 13, which gives V/V_{full}, Q/Q_{full}, and n/n_{full} as functions of y/D. The graphs in Figure 13 were created using values read from a similar graph in Steel & McGhee (reference # 2).

Equations for less than half full pipe flow: The diagram and equations below summarize the calculation of A, P, & R_h for a pipe flowing less than half full



Partially Full Pipe Flow Parameters
(Less Than Half Full)

$$r = D/2$$

$$h = y$$

$$\theta = 2\arccos\left(\frac{r - h}{r}\right)$$

$$A = \frac{r^2(\theta - \sin\theta)}{2}$$

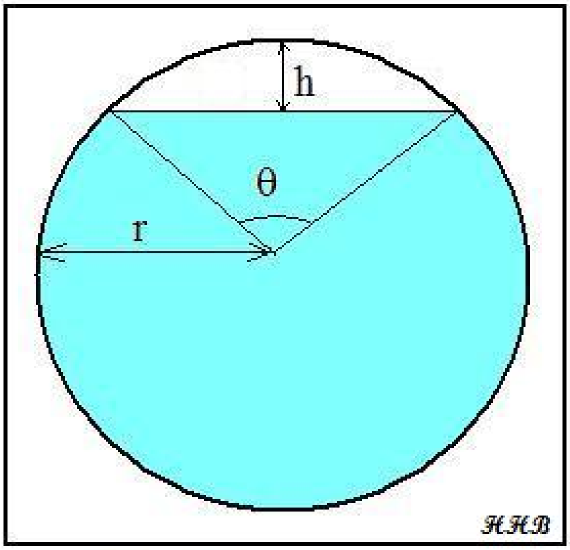
$$P = r * \theta$$

$$R_h = A/P$$

Figure 14. Diagram and Equations for Less Than Half Full Pipe Flow

For known pipe diameter, D, and depth of flow, y, the equations above allow calculation of cross-sectional area of flow, A, and wetted perimeter, P. Then the hydraulic radius can be calculated from $R_h = A/P$, for a pipe flowing less than half full.

Equations for more than half full pipe flow: The diagram and equations below summarize the calculation of A, P, & R_h for a pipe flowing more than half full:



Partially Full Pipe Flow Parameters
(More Than Half Full)

$$r = D/2$$

$$h = 2r - y$$

$$\theta = 2\arccos\left(\frac{r-h}{r}\right)$$

$$A = \pi r^2 - \frac{r^2(\theta - \sin\theta)}{2}$$

$$P = 2\pi r - r * \theta$$

$$R_h = A/P$$

Figure 15. Diagram and Equations for More Than Half Full Pipe Flow

Similarly, this set of equations allow calculation of A, P, and R_h if the pipe diameter and depth of flow are known for more than half full pipe flow.

Equation for n/n_{full}: As discussed above, in addition to the value of the hydraulic radius, the value of Mannings roughness coefficient is needed at the given y/D value in order to proceed with a Manning equation calculation.

The following equation can be used to calculate **n/n_{full}** as a function of **y/D**:

$$\frac{n}{n_{\mathrm{full}}} = 1 + \left(\frac{y}{D}\right)^{0.54} - \left(\frac{y}{D}\right)^{1.20}$$

The source for this equation is: Goswami, I., Civil Engineering All-in-One PE Exam Guide Breadth and Depth, 2nd Ed, McGraw-Hill, NY, NY, 2012, Equation 303.32

These equations provide all of the tools necessary to make Manning equation calculations for partially full pipe flow. With all of the equations and all of the steps required, Excel spreadsheets are ideal for this type of calculation. For a more detailed discussion of Manning equation calculations for partially full pipe flow, and the use of spreadsheets for those calculations, see reference #6 at the end of this course.

8. Uniform Flow Calculations for Natural Open Channels

The Manning equation is used a lot for natural channel flow calculations, as well as with the manmade channel examples we've already considered. One of the primary differences in using the Manning equation for natural channel flow is the lack of precision in estimation of Manning roughness coefficient values.

A. <u>The Manning Roughness Coefficient:</u> There are several approaches available for determining the Manning roughness coefficient, n, for flow in a natural open channel, including i) experimental determination of n; ii) use of a table or tables that give maximum, minimum and average n values for a variety of channel descriptions; and iii) a method devised by Cowan (reference #4) that uses a base n value determined by the general type of channel and modifies that base n value based on various descriptors of the channel. A bit more about each of these methods follows.

i) Experimental determination of the Manning roughness coefficient can be accomplished by measuring the depth of flowing water, the size and shape of the channel cross-section, and the volumetric flow rate for a given reach of channel. These measured values can be used to calculate an empirical value for n that can then be used for subsequent Manning equation calculations for that reach of channel.

Example #9: Calculate the Manning roughness coefficient, n, for a reach of river channel that has a bottom slope of 0.00028, with a cross-section that can be approximated as a trapezoid with bottom width equal to 8 ft and side slopes of horiz:vert = 4:1, if the flow rate has been estimated to be 75 cfs in that reach when the depth of water is 3 ¼ ft.

Solution: The Manning equation can be solved for n to give:

$$n = (1.49/Q)A(R_h^{2/3})S^{1/2}$$

The area and hydraulic radius of the trapezoidal cross-section are calculated as follows:

$$A = by + zy^2 = 8*3.25 + 4*3.25^2 = 68.25 \text{ ft}^2$$

$$P = b + 2y(1 + z^2)^{1/2} = 8 + 2*3.25(1 + 4^2)^{1/2} = 34.80 \text{ ft}$$

$$R_h = A/P = 68.25/34.80 = 1.96 \text{ ft}$$

Substituting values into the above equation for n gives:

$$n = (1.49/75)(68.25)(1.96^{2/3})(0.00028^{1/2}) = \underline{\mathbf{0.0355 = n}}$$

ii) There are **Tables of n values** in many textbooks and handbooks, as well as on websites. The table on the next two pages is an example from the Indiana Department of Transportation Design Manual (website ref # 1). Similar tables of n values are available on many state agency websites. The table below from the Indiana DOT Design Manual gives minimum, maximum, and normal values of the Manning roughness coefficient for a range of excavated or dredged and natural stream channels.

Type of Channel and Description	Minimum	Normal	Maximum
EXCAVATED OR DREDGED			
1. Earth, Straight and Uniform	0.016	0.018	0.020
a. Clean, recently completed	0.018	0.022	0.025
b. Clean, after weathering	0.022	0.025	0.030
c. Gravel, uniform section, clean	0.022	0.027	0.033
2. Earth, Winding and Sluggish			
a. No vegetation	0.023	0.025	0.030
b. Grass, some weeds	0.025	0.030	0.033
c. Dense weeds or aquatic plants in deep channel	0.030	0.035	0.040
d. Earth bottom and rubble sides	0.025	0.030	0.035
e. Stony bottom and weedy sides	0.025	0.035	0.045
f. Cobble bottom and clean sides	0.030	0.040	0.050
3. Dragline, Excavated or Dredged			
a. No vegetation	0.025	0.028	0.033
b. Light brush on banks	0.035	0.050	0.060
4. Rock Cut			
a. Smooth and uniform	0.025	0.035	0.040
b. Jagged and irregular	0.035	0.040	0.050
5. Channel Not Maintained, Weeds and Brush Uncut			
a. Dense weeds, high as flow depth	0.050	0.080	0.120
b. Clean bottom, brush on sides	0.040	0.050	0.080
c. Clean bottom, highest stage of flow	0.045	0.070	0.110
d. Dense brush, high stage	0.080	0.100	0.140
NATURAL STREAM			
1. Minor Stream (top width at flood stage < 100 ft)			
a. Stream on plain			
(1) Clean, straight, full stage, no rifts or deep pools	0.025	0.030	0.033
(2) Same as above, but more stones or weeds	0.030	0.035	0.040
(3) Clean, winding, some pools or shoals	0.033	0.040	0.045
(4) Same as above, but some weeds or stones	0.035	0.045	0.050
(5) Same as above, lower stages, more ineffective slopes and sections	0.040	0.048	0.055
(6) Same as (4), but more stones	0.045	0.050	0.060
(7) Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
(8) Very weedy reaches, deep pools, or floodway with heavy stand of timber and underbrush	0.075	0.100	0.150

NATURAL STREAM (contd.)			
Type of Channel and Description	**Minimum**	**Normal**	**Maximum**
1. Minor Stream (contd.)			
b. Mountain stream, no vegetation in channel, banks usually steep, trees and brush along banks submerged at high stages			
(1) Bottom: gravel, cobbles, and few boulders	0.030	0.040	0.050
(2) Bottom: cobbles with large boulders	0.040	0.050	0.07
2. Floodplain			
a. Pasture, no brush			
(1) Short grass	0.025	0.030	0.035
(2) High grass	0.030	0.035	0.050
b. Cultivated area			
(1) No crop	0.020	0.030	0.040
(2) Mature row crops	0.025	0.035	0.045
(3) Mature field crops	0.030	0.040	0.050
c. Brush			
(1) Scattered brush, heavy weeds	0.035	0.050	0.070
(2) Light brush and trees, in winter	0.035	0.050	0.060
(3) Light brush and trees, in summer	0.040	0.060	0.080
(4)Medium to dense brush, in winter	0.045	0.070	0.110
(5)Medium to dense brush, in summer	0.070	0.100	0.160
d. Trees			
(1) Dense willows, in summer, straight	0.110	0.150	0.200
(2) Cleared land with tree stumps, no sprouts	0.030	0.040	0.050
(3) Same as above, but with heavy growth of sprouts	0.050	0.060	0.080
(4) Heavy stand of timber, a few downed trees, little undergrowth, flood stage below branches	0.080	0.100	0.120
(5) Same as above, but with flood stage reaching branches	0.100	0.120	0.160
3. Major Stream (top width at flood stage > 100 ft). The *n* value is less than that for a minor stream of similar description, because banks offer less effective resistance.			
a. Regular section with no boulders or brush	0.025	n/a	0.060
b. Irregular and rough section	0.035	n/a	0.100

Table 3. Manning Roughness Coefficient, n, for Natural Channels

Example #10: What are the minimum, maximum, and normal values of the Manning roughness coefficient, n, for a minor mountain stream with no vegetation in the channel, banks usually steep, trees and brush along banks submerged at high stages, and cobbles with large boulders on the bottom, based on Table 3 below, from the Indiana DOT Design Manual?

Solution: From Table 3, the values of n for the described natural channel are:
$\mathbf{n_{min} = 0.040}$, $\mathbf{n_{max} = 0.07}$, $\mathbf{n_{normal} = 0.050}$

iii) The **Cowan procedure** was first presented in reference #4. There is also a good description of this method in McCuen (reference #5). This procedure uses a base n value with several terms added to it based on characteristics of the channel, as described below.

1. The **Base Roughness Coefficient, n_1,** is selected from the following based on the character of the channel:

- Channels in earth: $n_1 = 0.02$
- Channels cut into rock: $n_1 = 0.025$
- Channels in fine gravel: $n_1 = 0.024$
- Channels in coarse gravel: $n_1 = 0.028$

2. The value for the **Irregularity Modifier, n_2,** is selected from the following based on the degree of irregularity:

- Smooth (surface comparable to the best attainable for the materials involved) $n_2 = 0.000$
- Minor (good dredged channels; slightly eroded or scoured side slopes of canals) $n_2 = 0.005$

- Moderate (fair to poor deredged channels; moderately sloughed or eroded canal side slopes) $n_2 = 0.010$

- Severe (badly sloughed banks of natural streams; badly eroded or sloughed sides of canals or drainage channels; unshaped, jagged and irregular surfaces of channels excavated in rock $n_2 = 0.020$

3. The value for the **Cross Section Modifier, n_3,** is selected from the following based on the character of variations in size & shape of cross section:

- Change in size or shape occurs gradually $n_3 = 0.000$

- Large & small sections alternate occasionally or shape changes cause occasional shifting of main flow from side to side $n_3 = 0.005$

- Large & small sections alternate frequently or shape changes cause frequent shifting of main flow from side to side $n_3 = 0.010 – 0.020$

4. The value for the **Obstruction Modifier, n_4,** is selected from the following based on the relative effect of obstructions:

- Negligible $n_4 = 0.000$

- Minor $n_4 = 0.010 – 0.015$

- Appreciable $n_4 = 0.020 – 0.030$

- Severe $n_4 = 0.040 – 0.060$

5. The value for the **Vegetation Modifier, n_5,** is selected from the following based on the degree of vegetation effect on n:

- Low $n_5 = 0.005 – 0.010$
- Medium $n_5 = 0.010 – 0.020$
- Appreciable $n_5 = 0.020 – 0.050$
- Very High $n_5 = 0.050 – 0.100$

6. The value for the **Menadering Modifier, n_6,** is selected from the following based on the degree of meander:

- Minor (meander length**:**straight length = 1.0 – 1.2)
  $n_6 = 0.000$
- Appreciable (meander length**:**straight length = 1.2 – 1.5)
  $n_6 = 0.15n_s$
- Severe (meander length**:**straight length > 1.5)
  $n_6 = 0.30n_s$

Where $n_s = n_1 + n_2 + n_3 + n_4 + n_5$

7. The value of the Manning Roughness Coefficient is calculated from:

$$\mathbf{n = n_1 + n_2 + n_3 + n_4 + n_5 + n_6}$$

Example #11: Estimate the value for the Manning roughness coefficient for a channel in earth with minor irregularity, only gradual changes in size or shape, minor obstructions, medium effect of vegetation, and minor meander.

Solution: From the lists above: $n_1 = 0.02$, $n_2 = 0.005$, $n_3 = 0.000$, $n_4 = 0.010 – 0.015$, $n_5 = 0.010 – 0.020$, and $n_6 = 0.000$. Choosing the midpoint of the ranges given for n_4 and n_5 ($n_4 = 0.0125$ and $n_5 = 0.015$) gives the following equation for n:

$$n = 0.02 + 0.005 + 0.000 + 0.0125 + 0.015 + 0.000$$

$$\underline{\mathbf{n = 0.0525}}$$

B. <u>Manning Equation Calculations</u> for natural channels are the same as for manmade channels except for less precision in estimating the Manning roughness coefficient and greater difficulty in determining the hydraulic radius if the channel cross-section isn't a simple shape.

Example #12: A reach of channel of a minor stream on a plain is described as clean, winding, with some pools. The bottom slope is fairly constant at 0.00031 for this reach of channel. Its cross-section over this reach can be approximated as a trapezoid with a bottom width of 6 feet and side slopes of horiz**:**vert = 3:1. Find the range of flow rates that could be expected for this reach of channel for a 3.75 ft depth of flow, based on the maximum and minimum values of the Manning roughness coefficient from Table 3.

Solution: From the information given in the problem statement, b = 6 ft, y = 3.75 ft, z = 3, and S = 0.00031. From Table 3 (for the channel description given in the problem statement), $n_{min} = 0.033$ and $n_{max} = 0.045$.

From the equations for A, P, & R_h for a trapezoidal open channel:

$$R_h = [(6)(3.75) + 3(3.75^2)]/[6 + (2)(3.75)(1 + 3^2)^{1/2}] = 2.177 \text{ ft}$$

$$A = (6)(3.75) + 3(3.75^2) = 64.69 \text{ ft}^2$$

Substituting values into the Manning Equation [$Q = (1.49/n)A(R_h^{2/3})S^{1/2}$] gives the following results:

For n_{min} (0.033): $Q_{max} = (1.49/0.033)(64.69)(2.177^{2/3})(0.00031)^{1/2}$

$\underline{\mathbf{Q_{max} = 86.4\ ft^3/sec}}$

For n_{max} (0.045): $Q_{min} = (1.49/0.045)(64.69)(2.177^{2/3})(0.00031)^{1/2}$

$\underline{\mathbf{Q_{min} = 63.3\ ft^3/sec}}$

9. Summary

Open channel flow, which has a free liquid surface at atmospheric pressure, occurs in a variety of natural and man-made settings. Open channel flow may be classified as i) laminar or turbulent, ii) steady state or unsteady state, iii) critical, subcritical, or supercritical, and iv) uniform or nonuniform flow. Many practical cases of open channel flow can be treated as turbulent, steady state, uniform flow. Several open channel flow parameters are related through the empirical Manning Equation, for turbulent, uniform open channel flow ($\mathbf{Q = (1.49/n)A(R_h^{2/3})S^{1/2}}$). The use of the Manning equation for uniform open channel flow calculations and for the calculation of parameters in the equation, such as cross-sectional area and hydraulic radius, are illustrated in this course through worked examples for manmade channels and for natural channels.

10. References and Websites

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5. McCuen, R.H., *Hydrologic Analysis and Design*, 2nd Ed., Prentice Hall, Upper Saddle River, NJ, 1998.

6. Bengtson, H.H., "Partially Full Pipe flow Calculations with Excel Spreadsheets," an online article at www.engineeringexcelspreadsheets.com.

7. Bengtson, H.H., "The Manning Equation for Open Channel Flow Calculations", available as a paperback and as an Amazon Kindle e-book.

Websites:

1. Indiana Department of Transportation Design Manual, available on the internet at: http://www.in.gov/dot/div/contracts/standards/dm/2011/index.html

2. Illinois Department of Transportation Drainage Manual, available on the internet at: http://dot.state.il.us/bridges/brmanuals.html

3. Low Cost Easy to Use Excel Spreadsheets for Engineering Calculations, at www.engineeringexceltemplates.com.